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On Satellite Constellation Selection

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ON SATELLITE CONSTELLATION SELECTION

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ABSTRACT

Analytical estimates can be used to produce and discuss optimal constellations. They are in close agreement with phase-space estimates and exact solutions. They suggest that distributions of inclined orbits could reduce satellite numbers by factors of 2-3 while improving uniformity.

I. INTRODUCTION

In evaluating satellite constellations, an essential element is their ability to produce uniform distributions of satellites over the latitudes of interest, although practical considerations can be overriding. This report discusses some of the results available for evaluating constellations such as communications networks that require some degree of uniformity for practicality. Both computer simulations and analytic solutions are available. Numerical simulations are the most accurate, but analytic estimates give more insight into the sensitivity of results to parameter and missile distributions.

II. SATELLITE DISTRIBUTION

Satellite orbits can be taken to be circular. Their time and longitudinally averaged distributions are used below. A satellite whose orbit is inclined an angle $i \leq \pi/2$ from the north pole remains in a band $-i \leq \theta \leq i$ of latitude. If θ is the satellite's latitude and ϵ its azimuthal angle around its pole, from the spherical law of cosines

$$\sin\theta = \sin i \cdot \cos\epsilon, \tag{1}$$

where $\epsilon = 0$ for $\theta = i$. In traversing a latitude band $d\theta$ the satellite moves through an angle around its pole of[1]

$$d\epsilon = \cos\theta \cdot d\theta/(\sin^2 i - \sin^2\theta)^{1/2}, \tag{2}$$

traversing a fraction $d\epsilon/2\pi$ of its orbit in the process. This latitude band contains a fraction $\cos\theta \cdot d\theta/2$ of the earth's area. If the orbit contains 1 satellite, the satellite density is

$$n_+(\theta) = (d\epsilon/2\pi)/2\pi\cos\theta \cdot d\theta = 1/4\pi^2(\sin^2 i - \sin^2\theta)^{1/2}, \tag{3}$$

where distance is measured in multiples of the earth's radius R_e and n_+ is the density of satellites ascending in latitude. By symmetry, at any latitude there is an equal density, n_-, of satellites descending, and the total density is their sum[2]

$$n(\theta) = n_+ + n_- = 1/2\pi^2(\sin^2 i - \sin^2\theta)^{1/2}. \tag{4}$$

$n(\theta)$ is fairly flat, diverging integrably at $\theta = i$, but $n(\theta)$ is much larger at high than low latitudes.

Figure 1 shows the satellite densities as functions of latitude for constellations with single inclinations of 60, 70, 80, and 90 degrees. For polar orbits the satellites pile up over the pole. For smaller inclinations the variations are less, although the nonuniformities still approach factors of two at the northern limits of coverage.

As an aside, for single inclinations, satellites are moving in just two directions, ascending or descending, at any latitude. Since $n_+ = n_-$, their component densities are $n_+ = n_- = n(\theta)/2$ at each latitude. The angle between ascending orbits and the local east is

$$\Omega_+(\theta,i) = \sin^{-1}(d\theta/d\epsilon) = \sin^{-1}[(\sin^2 i - \sin^2\theta)^{1/2}/\cos\theta]. \tag{5}$$

For descending satellites the angle is $\Omega_- = -\Omega_+$. They cross at an angle of roughly $2\Omega_+$. At the orbit's maximum latitude $\Omega_+ = \Omega_-$

= 0; both ascending and descending orbits are tangent to the maximum latitude i there. At the equator, $\Omega_- = -i$, so the ascending satellites cross circles of constant latitude at monotonically decreasing angles.

III. CONSTELLATION OPTIMIZATION

This section uses the satellite distributions above to derive effective distributions of inclinations. A uniform distribution of satellites, $n_u = 1/4\pi$, for which the fraction of satellites between latitudes $0 \le \theta_1 \le \theta \le \theta_2$ is

$$f_u = \Sigma_{\theta 1}^{\theta 2}\, d\theta \cdot 2\pi\cos\theta \cdot (1/4\pi) = (\sin\theta_2 - \sin\theta_1)/2, \qquad (6)$$

which is the basis for comparison below.

A. Single Inclination

For a constellation with a single inclination i, Eq. (4) can be integrated to obtain the fraction of the satellites in any given latitude band, which is

$$f_i = \Sigma\, d\theta \cdot 2\pi\cos\theta / 2\pi^2(\sin^2 i - \sin^2\theta)^{1/2}. \qquad (7)$$

When the limits on the integral are θ_1 and θ_2, Eq. (7) gives

$$f_i = [\sin^{-1}(\sin\theta_2/\sin i) - \sin^{-1}(\sin\theta_1/\sin i)]/\pi. \qquad (8)$$

The average constellation concentration possible over θ_1 to θ_2 is $z \equiv f_i/f_u$, whose maximum occurs at $i = \theta_2$,[3] where it is

$$z = f_i/f_u = [1-(2/\pi)\sin^{-1}(\sin\theta_1/\sin\theta_2)]/(\sin\theta_2 - \sin\theta_1). \qquad (9)$$

Both n and z vary rapidly over high-latitude bands, which makes estimates sensitive to the averaging and specific values used.

B. Multiple Inclinations

The maximization of constellation concentration has been extended to multiple inclinations, which improve uniformity. The starting point is the generalization of Eq. (4) to multiple inclinations, which is

$$n(\theta) = \Sigma\, di \cdot N(i)/2\pi^2(\sin^2 i - \sin^2\theta)^{1/2}, \qquad (10)$$

where the integral is over the inclinations for which satellites can reach latitude θ. For a launch area within $\theta_1 \le \theta \le \theta_2$, the objective is to chose N(i) such that $n(\theta)$ is constant in that interval. The solution to this Abel equation is[4]

$$N(i) = \cos i \cdot \sin i / \sqrt{(\cos^2 i - \cos^2 \theta_2)} \quad (11)$$

for $\theta_1 \leq \theta \leq \theta_2$ and 0 elsewhere. For a uniform distribution over $\theta_1 = 0 \leq \theta \leq \theta_2 = \pi/2$ Eq. (11) reduces to $N(i) = \sin i$, for which Eq. (10) gives $n(\theta) = 1/4\pi$ for all θ and $\Sigma\, di \cdot N(i) = 1$, which is the geometric result for a random distribution of satellites.

The integral of $N(i)$ over $\theta_1 \leq i \leq \theta_2$ gives the fraction of the satellites in that interval. Dividing the geometric fraction from the uniform satellite distribution to that produced by the inclination distribution of Eq. (11) gives

$$z_w = 1/\sqrt{(\cos^2\theta_1 - \cos^2\theta_2)}. \quad (12)$$

This concentration factor is $z_w \approx 2.5$ for n uniform between $\theta_1 = 50^0$ and $\theta_2 = 60^0$, which is ≈ 20% less than the $z \approx 3$ from the non-uniform single inclination of Eq. (9).

The reduction in the number of satellites by using a distribution of inclinations rather than a single one is given by the ratio f_i/f_w. For coverage down to the equator $f_w \rightarrow 1/2$, $f_i \rightarrow \sin^{-1}(\sin\theta_2/\sin i)/\pi$. For $i = \theta_2 = \pi/2$, coverage to the pole, the ratio f_i/f_w is unity. For polar orbits $f_i \rightarrow \theta_2/\pi$. Thus, for $\theta_2 = \pi/4$, i.e., coverage of the U.S. and lower Europe down to the equator, $f_i \rightarrow 1/4$, and the gain for a distribution of inclinations is a factor of 2. Other combinations are shown in Fig. 2. For coverage up to 60^o, to include the Soviet Union, polar orbits would pay about a factor of 2 in satellites over a distribution up to 60^o. Thus, distributed constellations are significantly smaller as well as more uniform.

IV. SUMMARY AND CONCLUSIONS

Analytical estimates can be used to produce and discuss optimal constellation estimates for uniform coverage. Those estimates are in reasonable agreement with phase space estimates and exact solutions. They suggest that using distributions of inclined orbits could reduce the number of satellites by factors of 2-3, while improving uniformity.

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The author would like to acknowledge stimulating discussions of constellation scaling with Dr. Albert Petscheck and of constellation selection with Dr. Raymond Leopold of Motorola.

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Fig. 1 Satellite density vs inclination

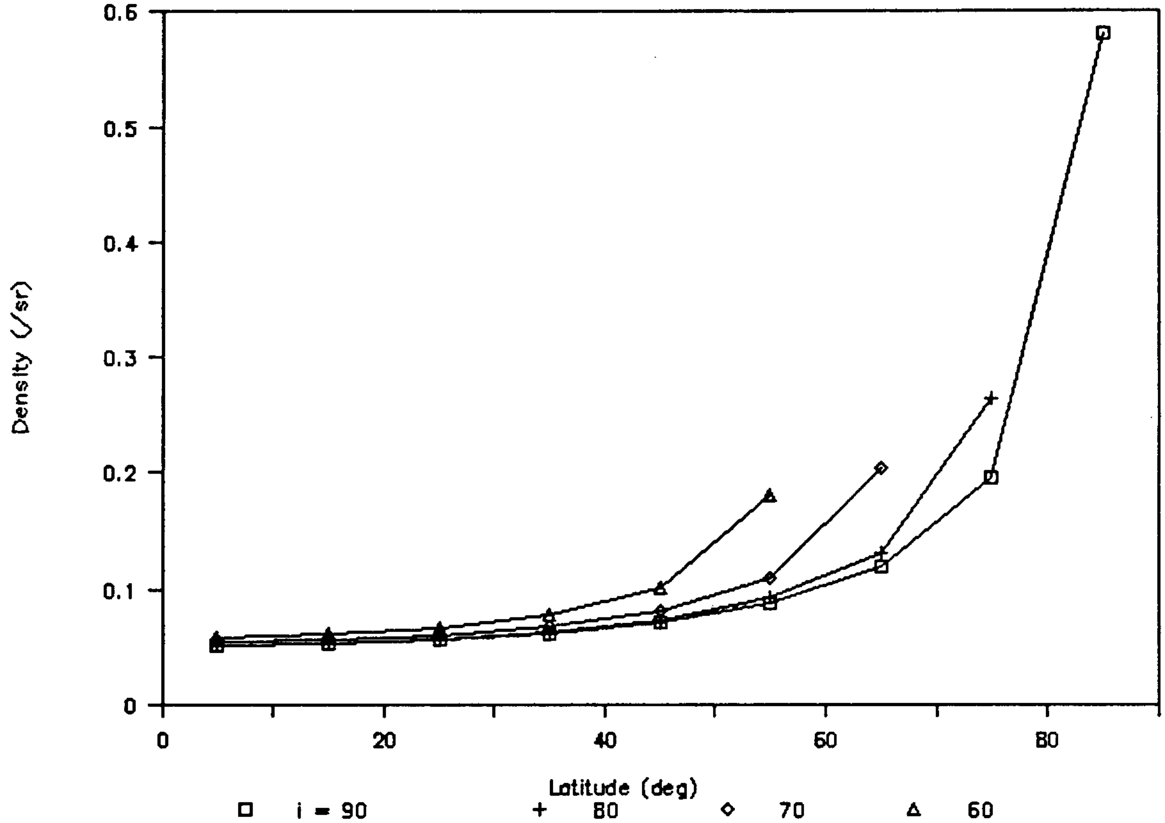

Fig. 2 Uniform vs inclined constel.